2-Layer Baroclinic Model

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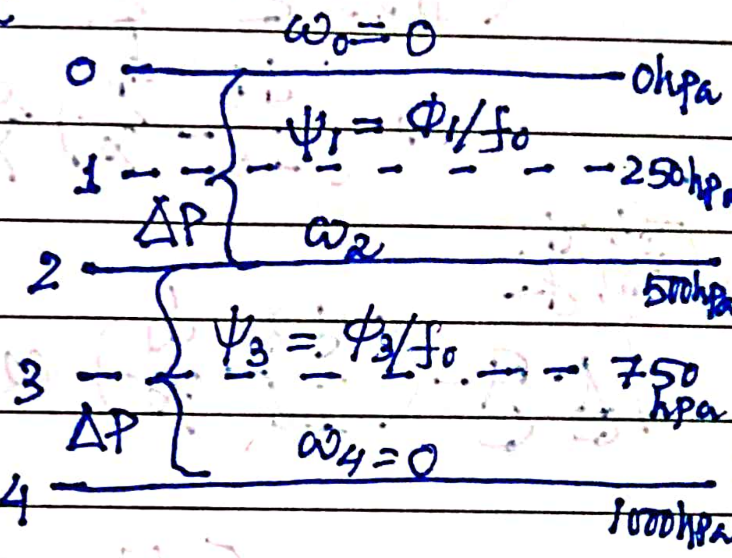
In 2 layer baroclinic model we ~~use~~ apply quasigeostrophic vorticity equation at level 1 & 3, as shown in the adjoining figure and thermodynamic energy equation at level 2.



Geopotential heights at level 1 & 3 will be denoted by Φ_1 & Φ_3 (not constants) and vertical velocity at levels 0, 2 & 4 are denoted as ω_0, ω_2 & ω_4.

In the 2-layer Baroclinic model, we use the boundary conditions, viz.,

$\omega_4 = 0$, $\omega_0 = 0$.

Thus applying quasigeostrophic vorticity equation at levels 1 & 3 gives:

$$\nabla^2\left(\frac{1}{f_0}\frac{\partial \Phi_1}{\partial t}\right) = -\frac{\hat{k}\times\vec{\nabla}\Phi_1}{f_0}\cdot\vec{\nabla}\left[\frac{\nabla^2\Phi_1}{f_0} + f\right] + \frac{f_0\,\omega_2}{\Delta P}$$

$$\Rightarrow \nabla^2\left(\frac{\partial \phi_1}{\partial t}\right) = \left(\hat{k}\times\vec{\nabla}\phi_1\right)\cdot\vec{\nabla}\left[\frac{\nabla^2\phi_1}{f_1}+f\right] + \frac{f_0^2\,\omega_2}{\Delta p} \quad \text{---} (1)$$

and

$$\nabla^2\left(\frac{\partial \phi_3}{\partial t}\right) = \left(\hat{k}\times\vec{\nabla}\phi_3\right)\cdot\vec{\nabla}\left[\frac{\nabla^2\phi_3}{f_0}+f\right] - \frac{f_0^2\,\omega_2}{\Delta p} \quad \text{--} (2)$$

Let us define ϕ_M & ϕ_T as following:

$$\phi_M = \frac{\phi_1+\phi_3}{2} \quad \& \quad \phi_T = \cancel{\phi_3 - \phi_T}\ \frac{\phi_1-\phi_3}{2}$$

So with these above notations, eq^n (1) & (2) reduces to:

$$\nabla^2 \frac{\partial}{\partial t}\left(\phi_M+\phi_T\right) = \hat{k}\times\vec{\nabla}\left(\phi_M+\phi_T\right)\cdot\vec{\nabla}\left[\frac{\nabla^2\left(\phi_M+\phi_T\right)}{f_0}+f\right] + \frac{f_0^2\,\omega_2}{\Delta p} \quad \text{---} (1(a))$$

$$\nabla^2 \frac{\partial}{\partial t}\left(\phi_M-\phi_T\right) = \hat{k}\times\vec{\nabla}\left(\phi_M-\phi_T\right)\cdot\vec{\nabla}\left[\frac{\nabla^2\left(\phi_M-\phi_T\right)}{f_0}+f\right] - \frac{f_0^2\,\omega_2}{\Delta p} \quad (2(a))$$

Adding 1(a) & 2(a) we get

$$2\nabla^2\left(\frac{\partial \phi_M}{\partial t}\right) = 2\hat{k}\times\vec{\nabla}(\phi_M).\vec{\nabla}\left[\frac{\nabla^2\phi_M}{f_0}\right]$$
$$+ 2\hat{k}\times\vec{\nabla}\phi_T.\vec{\nabla}\left(\frac{\nabla^2\phi_T}{f_0}\right)$$
$$+ 2\hat{k}\times\vec{\nabla}\phi_M.\vec{\nabla}f$$

$$\Rightarrow \nabla^2\left(\frac{\partial \phi_M}{\partial t}\right) = (\hat{k}\times\vec{\nabla}\phi_M).\vec{\nabla}\left(\nabla^2\frac{\phi_M}{f_0}\right)$$
$$+ (\hat{k}\times\vec{\nabla}\phi_T).\vec{\nabla}\left(\frac{\nabla^2\phi_T}{f_0}\right) + \beta\frac{\partial \phi_M}{\partial x} \quad \text{— 1(b)}$$

Similarly subtracting 2(a) from 1(a) we get,

$$\nabla^2\left(\frac{\partial \phi_T}{\partial t}\right) = (\hat{k}\times\vec{\nabla}\phi_M).\vec{\nabla}\left(\frac{\nabla^2\phi_T}{f_0}\right)$$
$$+ (\hat{k}\times\vec{\nabla}\phi_T).\vec{\nabla}\left(\frac{\nabla^2\phi_M}{f_0}\right) + \frac{f_0^2\omega_2}{\Delta p}$$
$$+ \beta\frac{\partial \psi_T}{\partial x} \quad \text{— 2(b)}$$

In the 2-layer Baroclinic model, thermo-dynamic energy equation (within QG-approximation) at level 2 is applied:

$$\frac{\partial}{\partial t}\left(\frac{\partial \phi}{\partial p}\right)_2 = -J + \sigma \omega_2 - \vec{V}_{g_2} \cdot \vec{\nabla}\left(\frac{\partial \phi}{\partial p}\right)_2 \quad \text{---}(3)$$

Where $J = \frac{R\dot{Q}}{PC_p} = \frac{\alpha}{C_p T}\dot{Q}$ = Diabatic heating term.

$\sigma = \frac{\alpha}{\theta}\frac{\partial \theta}{\partial p}$ = Static Stability Parameter.

& $\sigma < 0$ for statically stable.
$\sigma > 0$. for statically unstable.

$\vec{V}_{g_2}$ is the geostrophic wind at level 2, to be interpolated from that at level 1&3.

$$\therefore \vec{V}_{g_2} = \frac{\vec{V}_{g_1} + \vec{V}_{g_3}}{2} = \frac{\hat{k}}{f_0} \times \vec{\nabla}\frac{(\phi_1 + \phi_3)}{2}$$

$$= \frac{\hat{k} \times \vec{\nabla}\phi_M}{f_0}$$

Thus applying (3) at level 2 is fo

$$\frac{\partial}{\partial t}\left(\frac{\phi_3 - \phi_1}{\Delta p}\right) = -J + \sigma \omega_2 - \frac{\hat{k}}{f_0} \times \vec{\nabla}\phi_M \cdot \vec{\nabla}\left(\frac{\phi_3 - \phi_1}{\Delta p}\right) \quad \text{---}3(a)$$

$\Rightarrow$

$$-2\frac{\partial \phi_T}{\partial t} = -J\Delta P + \sigma \omega_2 \Delta P - \frac{\hat{k}\times\vec{\nabla}\phi_m}{f_0}\cdot\vec{\nabla}(-2\phi_T)$$

$$\Rightarrow \frac{\partial \phi_T}{\partial t} = +\frac{J\Delta P}{2} - \frac{\sigma\omega_2\Delta P}{2} - \left(\frac{\hat{k}\times\vec{\nabla}\phi_m}{f_0}\right)\cdot\vec{\nabla}\phi_T \quad \text{--- } 3(b)$$

Now, ω_2 is elliminated from 2(b) & 3(b)

For that we add $\lambda^2 \left(=\frac{2f_0^2}{\sigma(\Delta P)^2}\right)$ times 3(b) to 2(b), to obtain

$$(\nabla^2+\lambda^2)\left(\frac{\partial\phi_T}{\partial t}\right) = \left(\frac{\hat{k}\times\vec{\nabla}\phi_m}{f_0}\right)\cdot\vec{\nabla}(\nabla^2-\lambda^2)\phi_T + \left(\frac{\hat{k}\times\vec{\nabla}\phi_T}{f_0}\right)\cdot\vec{\nabla}(\nabla^2\phi_m) + \lambda^2\frac{J\Delta P}{2} + \beta\frac{\partial\psi_T}{\partial x} \quad \text{--- } (4)$$

Equations 1(b) & (4) may be treated as model equation for 2-layer Baroclinic model.

Algorithims for predicting ψ_1 & ψ_3

i> observation of GP at 250 & 750 hpa.

ii> Analysis & initialization of GP at 250 & 750 hpa. Compute $\frac{\partial \phi}{\partial p}$ at each point at 500 hpa.

iii> Compute ϕ_M & ϕ_T at each point. ~~on 500 hpa~~

iv> Compute $\frac{\partial \phi_M}{\partial x}$, $\frac{\partial \phi_M}{\partial y}$, $\frac{\partial \phi_T}{\partial x}$ at each point.

v> Compute Laplaceans of ϕ_M & ϕ_T at each point.

vi> Compute Jacobians, using Arakawa Jacobean $J(\phi_M, \nabla^2\phi_M)$, $J(\phi_T, \nabla^2\phi_T)$, $J(\phi_T, \nabla^2\phi_M)$, $J(\phi_M, (\nabla^2-\lambda^2)\phi_T)$

vii> Compute $\lambda^2 J \Delta P$ at each point

viii> Then RHS of 1(b) & 4 are completely known.

ix> Solve the Poisson's eq^n 1(b) & 4 using Relaxation method for $\frac{\partial \psi_M}{\partial t}$ and $\frac{\partial \psi_T}{\partial t}$ at a time level, say, $t = t_0$.

x) Using forward difference

$$\phi_M(t_0+\Delta t) = \psi_M(t_0) + \Delta t\, C_{ij}^{t_0}$$

$$\phi_T(t_0+\Delta t) = \psi_T(t_0) + \Delta t\, D_{ij}^{t_0}$$

where $C_{ij}^{t_0} = \left(\frac{\partial \phi_M}{\partial t}\right)_{i,j}^{t=t_0}$ &

$D_{ij}^{t_0} = \left(\frac{\partial \phi_T}{\partial t}\right)_{i,j}^{t=t_0}$ are

the solutions of the Poisson's equations at an arbitrary grid point (i,j) at time $= t_0$, for $\frac{\partial \psi_M}{\partial t}$ & $\frac{\partial \psi_T}{\partial t}$ respectively.

Then $\phi_1(t_0+\Delta t) = \underline{\phi_M(t_0+\Delta t) + \phi_T(t_0+\Delta t)}$

$$\phi_3(t_0+\Delta t) = \phi_M(t_0+\Delta t) - \phi_T(t_0+\Delta t)$$

Can be found out at each grid point.

~~The~~ xi) The above steps are repeated to find out ϕ_1 & ϕ_3 at any future time steps.